MATH 1100 DEFINITIONS, FORMULAS AND PROCEDURES

I. Factoring.

$x^2 - a^2 .= (x-a)(x+a)$,
$(x+a)^2 = x^2 + 2xa + a^2$
$(x-a)^2 = x^2 - 2xa + a^2$,
$x^3 - a^3 = (x-a)(x^2 + xa + a^2)$
$x^3 + a^3 = (x+a)(x^2 - xa + a^2)$

II. Exponents

$a^0 = 1 \qquad a^x a^y = a^{x+y}$
$(a^x)^y = a^{xy} \qquad a^{-x} = \frac{1}{a^x}$
$(ab)^x = a^x b^x \qquad (\frac{a}{b})^x = \frac{a^x}{b^x}$

III. Radicals And Fractional Exponents

If n is even and $a \geqq 0$ then $\sqrt[n]{a} = b$ if $b^n = a$
If n is odd and then $\sqrt[n]{a} = b$ if $b^n = a$
In both cases $\sqrt[n]{a}$ is called the principal nth-root of a
If n is odd and then $\sqrt[n]{a^n} = a$
If n is even and then $\sqrt[n]{a^n} = |a|$
$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$
$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
$a^{\frac{1}{n}}$ means $\sqrt[n]{a}$ or
$a^{\frac{m}{n}}$ means $\sqrt[n]{a^m}$ or $(\sqrt[n]{a})^m$
$a^{-\frac{m}{n}}$ means $\frac{1}{a^{\frac{m}{n}}}$

IV. Quadratic Formula, Distance, midpoint and Circles.

If $ax^2 + bc + c = 0$, with $a \neq 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Given two points, (x_0, y_0) and (x_1, y_1)
the **distance** between the points is $\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$,
and the **midpoint** is $(\dfrac{x_1 + x_0}{2}, \dfrac{y_1 + y_0}{2})$,(average the coordinates).
The equation of a **circle** with center (h, k) and radius $r > 0$ is $(x-h)^2 + (y-k)^2 = r^2$.

V. Lines **Lines and slope.**

Lines not parallel to y axis

Given two points, (x_0, y_0) and (x_1, y_1) with $x_0 \neq x_1$
(line determined by (x_0, y_0) and (x_1, y_1) **not parallel to y axis**)
the **slope** of the line between the two points is
$\dfrac{y_1 - y_0}{x_1 - x_0} = \dfrac{rise}{run} = m$
and the **point-slope** equation of the line
through (x_0, y_0) and (x_1, y_1) is $y - y_0 = m(x - x_0)$.
The **slope-intercep**t equation of the line through
(x_0, y_0) and (x_1, y_1) is $y = mx + b$ with
$(0, b)$ **the** y **intercept.**
Two lines are **parallel** if they have the same slope ($\mathbf{m}_1 = \mathbf{m}_2$) and two lines are **perpendicular** if one slope is the negative reciprocal of the other $\mathbf{m}_1 = -\dfrac{1}{m_2}$

Lines not parallel to y axis

.If $x_0 = x_1$,(line determined by (x_0, y_0) and (x_1, y_1) **parallel to y axis**) the slope is undefined the equation of the line through (x_0, y_0) and (x_1, y_1) is $x = x_0$.

VI. The Domain, Range And Function Composition

The domain of a function is where the function is defined..
If the domain is not specified it is taken to be those real numbers for which the function makes sense.The range is the set of image points i.e. the set of all $f(x)$ where x is in the domain, Also if the graph G is known then
the domain of f = the projection of G on the X axis and
the range of f = the projection of G on the Y axis
$(fog)(x)$ means $f(g(x))$ and $(gof)(x)$ means $g(f(x))$

VII. Point Symmetry And Set Symmetry

Given a point (x, y), the **reflection** of this point about the **y-axis is** $(-x, y)$,the **reflection** of this point about the **origin** is $(-x, -y)$,and the **reflection** of this point about the **x-axis** is $(x, -y)$.A set of points G
(usually the graph of an equation) is called **symmetric abou**t the **y-axis** if G is the same as its reflection about the y-axis. AA set of points G
(usually the graph of an equation)
is called **symmetric about the origin**
if G is the same as its
reflection about the origin. A set of points G is called **symmetric about the x-axis**. If G is the same as its reflection about the x-axis.

VIII. Functions And Symmetry

If the graph G is that of a function $f(x)$ the graph G will be symmetric about the y-axis if and only if
$f(x) = f(-x)$, i.e. an **even** function.
If the graph G is that of a function $f(x)$ the graph G will be symmetric about the origin if and only if
$-f(x) = f(-x)$, or equivalently $f(x) = -f(-x)$
and is called an **odd** function.

IX. Increasing And Decreasing functions

$f(x)$ be is increasing on $(a.b)$ if whenever $a < t < u < b$ then $f(t) < f(v)$i.e. the graph of if slants "up on $(a.b)$"
$f(x)$ be is decreasing on $(a.b)$ if whenever $a < t < u < b$ then $f(t) > f(v)$ i.e. the graph of if slants "down on $(a.b)$"

X. Shifting

If the graph of a function is G we geometrically shift G in natural ways and change the function so that the modified function gives the shift..
up a units : $function + a$.
down a units ; $function - a$
left a units : evaluate the function at $x + a$;(replace x with $x + a$)
right a units evaluate the function at $x - a$ (replace x with $x - a$)
Reflect about $y - axis$; ; evaluate the function at $-x$;
i.e. replace x with $-x$
Reflect about the $x - axis$; ; negate the function i.e. replace the $function$ with $-function$

XI.. Average Value(AV) (Or Difference Quotient)(DQ) of a Function

Let $f(x)$ be defined on $[a.b]$ The **AV** of $f(x)$ on $[a.b]$ is
$\dfrac{f(b) - f(a)}{b - a}$
Let $f(x)$ be defined on $[x, x+h]$ The **DQ** of $f(x)$ on $[x, x+h]$ is
$\dfrac{f(x+h) - f(x)}{h}$

XII..Inverse Functions
If $y = f(x)$ is a one to one function then it has an inverse $f^{-1}(x)$
To find the inverse solve for x in terms of y to get an expression
for y and then in the expression for y replace y with x to get $f^{-1}(x)$. Also The domain of $f(x)$ = the range of $f^{-1}(x)$ and The domain of $f^{-1}(x)$ = the range of $f(x)$
The graph of $f^{-1}(x)$ may be obtained rotating the graph of f
about the line $y = x$ (180 degree perpendicular rotation)
XIII. Quadratics And Parabolas
If $y = f(x) = ax^2+bc+c$ with $a \neq 0$. The graph is a **parabola**
If $a > 0$ the **parabola** opens up
If $a < 0$ the **parabola** opens down
The **Vertex** is $(\frac{-b}{2a}, f(\frac{-b}{2a}))$
The **Axis of Symmetry** is $x = \frac{-b}{2a}$
If $y - k = d(x-h)^2$ then (h, k) is the vertex,
and $x = h$ is the **Axis of Symmetry**
XIV. Finding The Roots (or Zeros) of a Polynomial
If $P(x) = a_n x^n + ...a_0$ is a polynomial with integer coefficients
then any rational root of $P(x)$ must have the form $\frac{\text{a factor of } a_0}{\text{a factor of } a_n}$
If $P(x)$ is a polynomial Then a is a root of $P(x)$ (i.e. $P(a) = 0$)
if and only if $x - a$ divides $P(x)$.
Use the above two techniques to completely factor $P(x)$
XV. Vertical(VA), Horizontal(HA),
And Slant (SA) Asymtopes.
Let $R(x) = \frac{P(x)}{Q(x)}$ be rational (Must Be Put in Lowest terms!). Then
1. x = any root (zero) of Q(x) is a VA
2.If Deg P(x) < Deg Q(x) then y=0 is a HA
If Deg P(x) > Deg Q(x) then there is no HA
If Deg P(x) = Deg Q(x) then $y = \frac{\text{leading coeficent of P}}{\text{leading coeficent of Q}}$ is an HA
3.A SA will exist only if Deg P(x) = Deg Q(x)+1 and then in
this case divide P by Q to obtain a line which is the SA
XVI. Solving Polynomial and Rational Inequalities
Solving Polynomial Inequalities
Let P(x) and Q(x) be polynomials. To solve
$P(x)$ ($\leqq, <, \geqslant$, or $>$) $Q(x)$. Procedure
1. Let T(x) = P(x) - Q(x) .
2, Find the roots of T(x) and plot them on the real line
3, The roots naturally. divide the line onto parts and on the inside of each part T(x) is always positive or negative
4. Choose a point z on the inside of each part and compute T(z) (which is + or -) The Parts that are positive form a solution to $P(x) > Q(x)$. and The Parts that are negative form a solution to $P(x) < Q(x)$. The solutions to $P(x) \geqslant Q(x)$
and $P(x) \leqq Q(x)$ are the same except include the real endpoints of each part.
Solving Rational Inequalities
Let P(x) and Q(x) be Rational Functions. To solve
$P(x)$ ($\leqq, <, \geqslant$, or $>$) $Q(x)$. Procedure
1. Let T(x) = P(x) - Q(x) = $\frac{\text{numerator}}{\text{denominator}}$
2. Put T(x) = $\frac{\text{numerator}}{\text{denominator}}$ in lowest terms
3. Find the roots of the top and bottom and plot them on the
real line
4 The roots naturally divide the line onto parts and on the inside of each part T(x) is always positive or negative
5. Choose a point z on the inside of each part and compute T(z) (which is + or -)
The Parts that are positive form a solution to $P(x) > Q(x)$. and
The Parts that are negative form a solution to $P(x) < Q(x)$
The solutions to $P(x) \geqslant Q(x)$ and $P(x) \leqq Q(x)$ are the same
except check the real endpoints of each part as some endpoints may be included.
XVII. **Direct , Inverse And Mixed Proportionality**
y is **directly proportional** to x if $y = kx$ for some $k > 0$
y is **inversely proportional** to x if $y = k\frac{1}{x}$ for some $k > 0$
The above concepts can be combined for two or more variables to yield the concept of **Mixed Proportionality** with the specifics made clear from problem context,
XVIII. Logs
$\log_b a = c$ means $b^c = a$ where b is the base
$\log(Z)$ is only defined if $Z > 0$
$\log(xy) = \log(x) + \log(y)$ (any base)
$\log(x^p) = p\log(x)$ (any base)
$\log(\frac{x}{y}) = \log(x) + \log(y)$ (any base)
$\log_b b^z = z$
$b^{\log_b z} = z$
$\ln x$ means $\log_e x$
$\ln e^x = z$ (very useful for solving exponential equations)
$e^{\ln x} = z$ (very useful for solving logarithmic equations)
Note: ln and $\log_{10}$ = log are in your calculators. for other bases b use
$\log_b a = \frac{\ln a}{\ln b} = \frac{\log a}{\ln b}$ (Change of base)
XIX. Money And Interest Rates
Period or Complex Compounding
A = amount at end of investment period, P= principal, r = yearly simple interest rate, t = time invested and n = number of investment periods. Then
$A = P(1 + \frac{r}{n})^{nt}$
Continuous Compounding $A = Pe^{rt}$